Vi får:

$$y_{ijk} = \bar{y}_{\cdots} + \underbrace{\bar{y}_{i\cdot\cdot} - \bar{y}_{\cdots}}_{\hat{\alpha}_i} + \underbrace{\bar{y}_{\cdot j\cdot} - \bar{y}_{\cdots}}_{\hat{\beta}_j} + \underbrace{\bar{y}_{ij\cdot} - \bar{y}_{i\cdot\cdot} - \bar{y}_{\cdot j\cdot} + \bar{y}_{\cdots}}_{(\hat{\alpha\beta})_{ij}} + y_{ijk} - \bar{y}_{ij\cdot}$$

Vi har: $$\sum_i \sum_j \sum_k (\bar{y}_{ij\cdot} - \bar{y}_{\cdots})(y_{ijk} - \bar{y}_{ij\cdot}) = 0$$

som gir:

$$\sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} (y_{ijk} - \bar{y}_{\cdots})^2 = \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} (\bar{y}_{i\cdot\cdot} - \bar{y}_{\cdots})^2 + \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} (\bar{y}_{\cdot j\cdot} - y_{\cdots})^2$$
$$+ \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} (\bar{y}_{ij\cdot} - \bar{y}_{i\cdot\cdot} - \bar{y}_{\cdot j\cdot} + y_{\cdots})^2 + \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} (y_{ijk} - \bar{y}_{ij\cdot})^2$$

eller $$SS_T = SS_A + SS_B + SS_{AB} + SS_E$$

Under H_0 har vi:

$$E\left[\frac{SS_A}{a-1}\right] = \sigma^2$$

$$E\left[\frac{SS_B}{b-1}\right] = \sigma^2$$

$$E\left[\frac{SS_{AB}}{(a-1)(b-1)}\right] = \sigma^2$$

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$$E\left[\frac{SS_E}{ab(n-1)}\right] = \sigma^2$$

The analysis of variance table

Sources	SS	DF	MS	F
A	$SS_A = nb \sum_{i=1}^{a} (\bar{y}_{i..} - \bar{y}_{...})^2$	$a-1$	$SS_A/(a-1)$	$F = \frac{MSA}{MSE}$
B	$SS_B = na \sum_{j=1}^{b} (\bar{y}_{.j.} - \bar{y}_{...})^2$	$b-1$	$SS_B/(b-1)$	$F = \frac{MSB}{MSE}$
Interaction (AB)	$SS_{AB} = n \sum_{i=1}^{a} \sum_{j=1}^{b} (\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...})^2$	$(a-1)(b-1)$	$SS_{AB}/(a-1)(b-1)$	$F = \frac{MSAB}{MSE}$
Error	$SS_E = \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} (y_{ijk} - y_{ij.})^2$	$ab(n-1)$	$\frac{SS_E}{ab(n-1)}$	

Before testing for an eventual significance of main effects, we should always test if there are interactions. If there are interactions between the factors, it means that the effect of at least one level of factor A is dependent on the level of factor B. For such situations an interaction plot will be useful in order to interpret the level of the factors.

An interaction plot is a plot of $\bar{Y}_{ij}$ against the level of factor B (or A). If there are no significant interactions, one interpret the effects of the factors in a normal way. It is usual to use

$$\frac{SSE}{ab(n-1)}$$ as an estimate for the variance of the error

in any case.

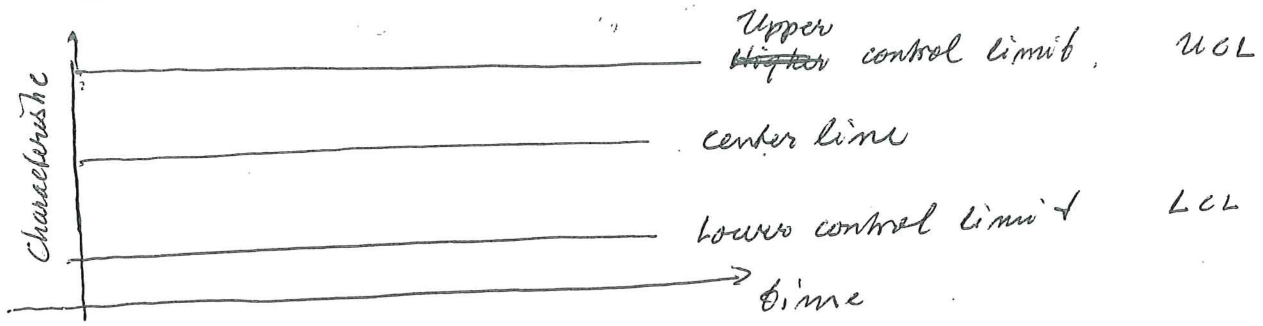
Chap 17. Statistical Process control / Quality control.

Purpose: To ensure that the performance of a process is maintaining an acceptable level of quality.

Idea. If all steps in a production process are in control we should avoid producing scrap.

Producing scrap is as costly as producing products with good quality.

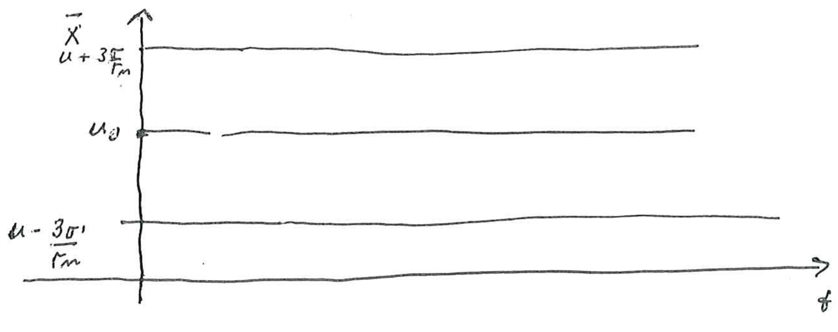
Tool. Control chart.



At each time t we take a random sample, $x_1, \ldots, x_n$, compute a statistic and decide if the statistic is smaller than LCL or larger than UCL. If that happens the process is out of control. If not, it is in control. Example of sequential hypothesis testing.

Example. Random sample, $X_1, \ldots, X_n$ at time t. $X_i \sim N(\mu_0, \sigma^2), i=1,2,\ldots,n$

Then $\bar{X} \sim N(\mu_0, \frac{\sigma^2}{n})$ and a typical control chart would be



Now suppose the process is in control and let α ~~be~~ be the probability of a false alarm. α is the type I error.

Let T_c be the time until a false alarm is detected for the first time.

$T_c \sim$ geometrical distributed with probability α

and, $E[T_c] = \dfrac{1}{\alpha}$

If $= \mu = \mu_0$ $(h=0)$, we have

$$\alpha = P\left(\bar{X} \geq \mu_0 + \frac{k\sigma}{\sqrt{n}} \,\Big|\, \mu=\mu_0\right) + P\left(\bar{X} \leq \mu_0 - \frac{k\sigma}{\sqrt{n}} \,\Big|\, \mu=\mu_0\right)$$

$$= 2P\left(\frac{\bar{X}-\mu_0}{\frac{\sigma}{\sqrt{n}}} \leq -k \,\Big|\, \mu=\mu_0\right) = 2\,\Phi(-k)$$

With $k=3$ $\quad 2\,\Phi(-3) = 0{,}0026 = \alpha$.

and $E[T_c) = \dfrac{1}{0{,}026} = 385$, i.e. the expected time between false alarms is 385 time units.

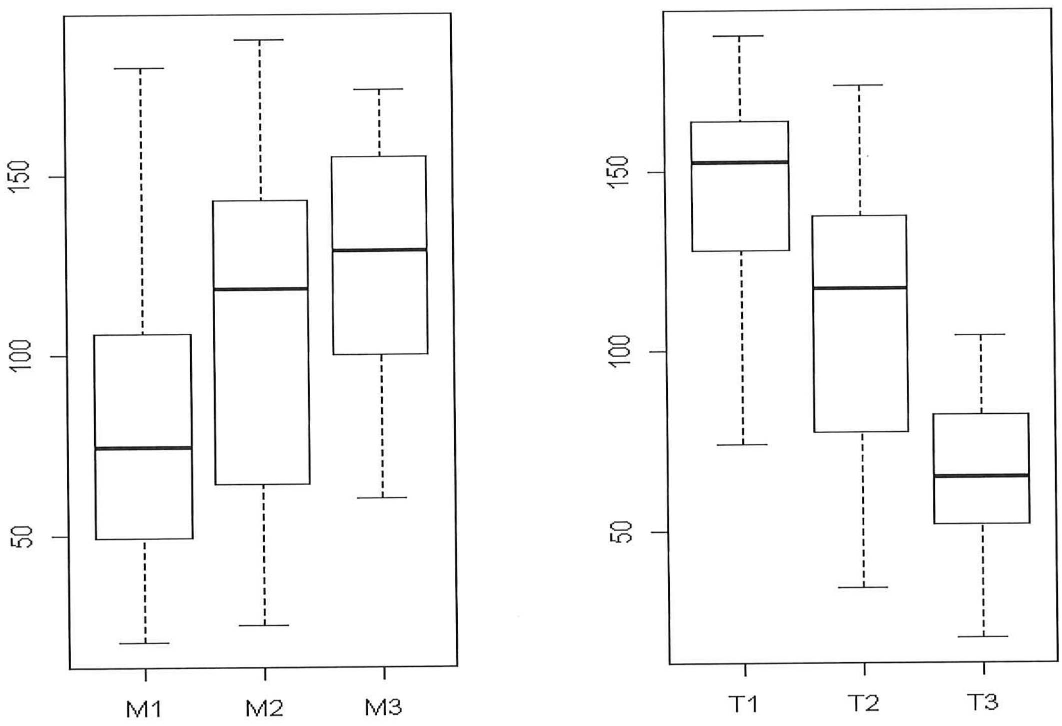
Control charts is a way to separate between natural variation (~~natural~~ natural error) and assignable causes (something has happened)

The Battery example

Testing lifetime of batteries

		Temperature		
		50°C	65°C	85°C
Material	M1	130 155 74 180	34 40 80 75	20 70 82 58
	M2	150 188 159 126	136 122 106 115	25 70 58 45
	M3	138 110 168 160	174 120 150 139	96 104 82 60

Boxplots



Cell averages

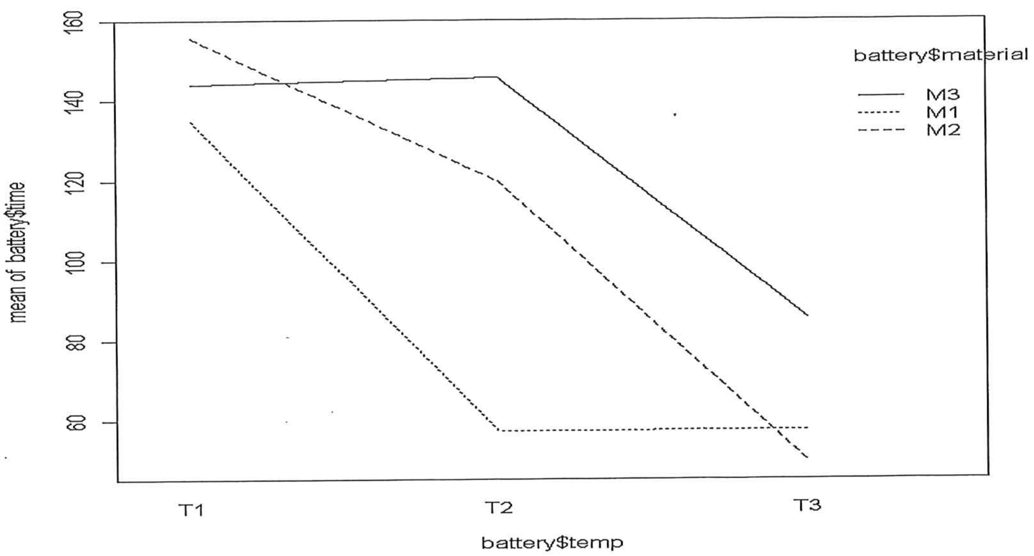
	T1	T2	T3
M1	134.75	57.25	57.5
M2	155.75	119.75	49.5
M3	144.00	145.75	85.5

Analysis of variance table

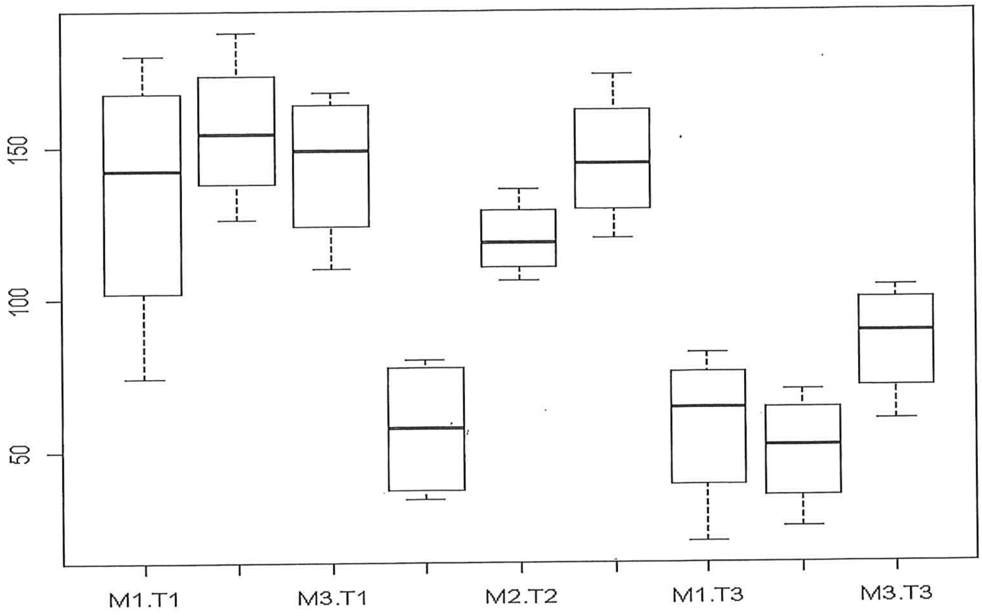
Response: time

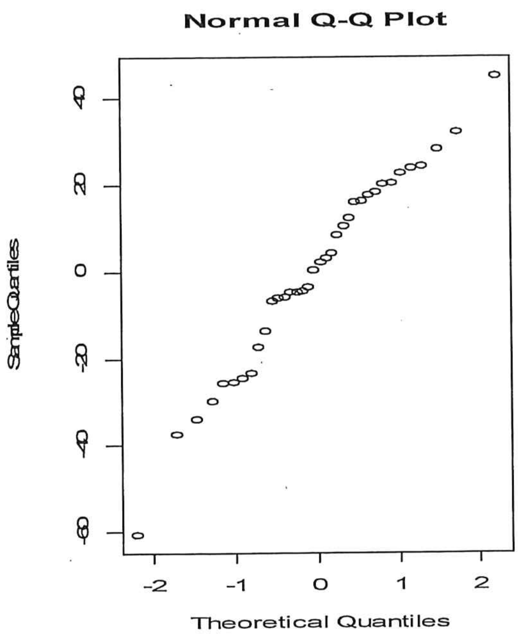
	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
material	2	10684	5341.9	7.9114	0.001976	**
temp	2	39119	19559.4	28.9677	1.909e-07	***
material:temp	4	9614	2403.4	3.5595	0.018611	*
Residuals	27	18231	675.2			

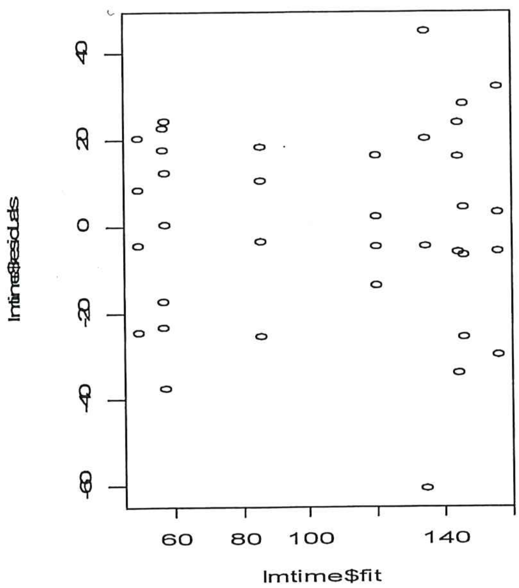
Interaction plot



Cell Boxplots







Box-plot of residuals